M11/5/MATHL/HP3/ENG/TZ0/DM/M

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MARKSCHEME

May 2011

MATHEMATICS
DISCRETE MATHEMATICS

Higher Level

Paper 3

10 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Write the marks in red on candidates' scripts, in the right hand margin.

- Show the **breakdown** of individual marks awarded using the abbreviations ***M1***, ***A1***, *etc.*
- Write down the total for each **question** (at the end of the question) and **circle** it.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.
- Once a correct answer to a question or part-question is seen, ignore further working.

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

*Implied marks appear in **brackets e.g. (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the answer (i.e. there is no working expected), then **FT** marks should be awarded if appropriate.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). Apply a **MR** penalty of 1 mark to that question. Award the marks as usual and then write –1(**MR**) next to the total. Subtract 1 mark from the total for the question. A candidate should be penalized only once for a particular mis-read.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. The mark should be labelled **(d)** and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

Unless the question specifies otherwise, ***accept*** *equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

$$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(=10\cos(5x-3)\right) \qquad \textbf{\textit{A1}}$$

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.

- **Rounding errors**: only applies to final answers not to intermediate steps.
- **Level of accuracy**: when this is not specified in the question the general rule applies: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

Candidates should be penalized ***once only IN THE PAPER*** *for an accuracy error* ***(AP)****. Award the marks as usual then write* ***(AP)*** *against the answer. On the* ***front*** *cover write –1(****AP****). Deduct 1 mark from the total for the paper, not the question*.

- If a final correct answer is incorrectly rounded, apply the ***AP***.
- If the level of accuracy is not specified in the question, apply the ***AP*** for correct answers not given to three significant figures.

If there is no working shown, and answers are given to the correct two significant figures, apply the ***AP***. However, do **not** accept answers to one significant figure without working.

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (e.g. TI-89) are not allowed.

Calculator notation

The Mathematics HL guide says:

Students must always use correct mathematical notation, not calculator notation.

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

1. (a) $315 = 5 \times 56 + 35$ ***M1***

$56 = 1 \times 35 + 21$

$35 = 1 \times 21 + 14$ ***A1***

$21 = 1 \times 14 + 7$

$14 = 2 \times 7$ ***A1***

therefore gcd $= 7$ ***A1***

[4 marks]

(b) (i) $7 = 21 - 14$ ***M1***

$= 21 - (35 - 21)$

$= 2 \times 21 - 35$ ***(A1)***

$= 2 \times (56 - 35) - 35$

$= 2 \times 56 - 3 \times 35$ ***(A1)***

$= 2 \times 56 - 3 \times (315 - 5 \times 56)$

$= 17 \times 56 - 3 \times 315$ ***(A1)***

therefore $56 \times 51 + 315 \times (-9) = 21$ ***M1***

$x = 51,\ y = -9$ is a solution ***(A1)***

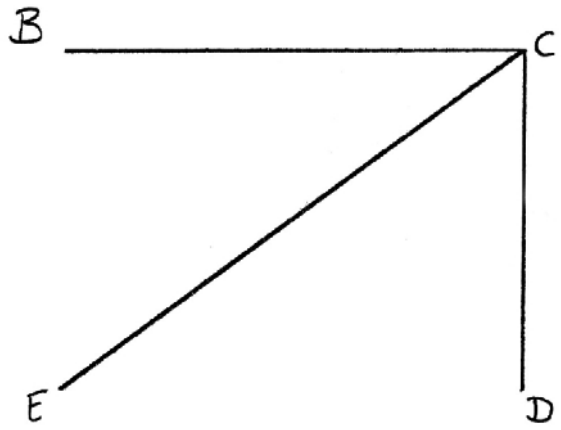
the general solution is $x = 51 + 45N,\ y = -9 - 8N,\ N \in \mathbb{Z}$ ***A1A1***

(ii) putting $N = -2$ gives $y = 7$ which is the required value of x ***A1***

[9 marks]

Total [13 marks]

2. (a) using any method, the minimum spanning tree is ***(M1)***



A2

Note: Accept MST = {BC, EC, DC} or {BC, EB, DC}

Note: In graph, line CE may be replaced by BE.

lower bound = weight of minimum spanning tree + 2 smallest weights connected to A ***(M1)***

$= 11 + 13 + 14 + 10 + 15 = 63$ ***A1***

[5 marks]

continued …

Question 2 continued

(b) weight of ADCBEA $= 10 + 14 + 11 + 13 + 15 = 63$ ***A1***

[1 mark]

(c) the conclusion is that ADCBEA gives a solution to the travelling salesman problem ***A1***

[1 mark]

Total [7 marks]

3. (a) $a = \lambda c + 1$ ***M1***

so $ab = \lambda bc + b \Rightarrow ab \equiv b \pmod{c}$ ***A1AG***

[2 marks]

(b) the result is true for $n = 0$ since $9^0 = 1 \equiv 1 \pmod{4}$ ***A1***

assume the result is true for $n = k$, *i.e.* $9^k \equiv 1 \pmod{4}$ ***M1***

consider $9^{k+1} = 9 \times 9^k$ ***M1***

$\equiv 9 \times 1 \pmod{4}$ **or** $1 \times 9^k \pmod{4}$ ***A1***

$\equiv 1 \pmod{4}$ ***A1***

so true for $n = k \Rightarrow$ true for $n = k + 1$ and since true for $n = 0$ result follows by induction ***R1***

Note: Do not award the final ***R1*** unless both ***M1*** marks have been awarded.

Note: Award the final ***R1*** if candidates state $n = 1$ rather than $n = 0$

[6 marks]

(c) let $M = (a_n a_{n-1} \ldots a_0)_9$ ***(M1)***

$= a_n \times 9^n + a_{n-1} \times 9^{n-1} + \ldots + a_0 \times 9^0$ ***A1***

EITHER

$\equiv a_n \pmod{4} + a_{n-1} \pmod{4} + \ldots + a_0 \pmod{4}$ ***A1***

$\equiv \sum a_i \pmod{4}$ ***A1***

so M is divisible by 4 if $\sum a_i$ is divisible by 4 ***AG***

OR

$= a_n(9^n - 1) + a_{n-1}(9^{n-1} - 1) + \ldots + a_1(9^1 - 1)$

$+ a_n + a_{n-1} + \ldots + a_1 + a_0$ ***A1***

Since $9^n \equiv 1 \pmod{4}$, it follows that $9^n - 1$ is divisible by 4, ***R1***

so M is divisible by 4 if $\sum a_i$ is divisible by 4 ***AG***

[4 marks]

Total [12 marks]

4. (a) (i) any Hamiltonian circuit ACBEFDA ***A2***

(ii) no Eulerian circuit exists because the graph contains vertices of odd degree ***A2***

[4 marks]

(b) (i) the adjacency matrix is

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$ ***A2***

Note: Award ***A2*** even if the zero is replaced by dashes.

(ii) to find the number of walks of length 5, we need to raise this matrix to the 5^{th} power ***M1***
doing this, we find that the smallest off-diagonal term corresponds to EF ***A1***

(iii) any 4 walks, for example:
EBEBEF
EDEDEF
EFEFEF
EBADEF ***A2***

Note: Award ***A1*** for 3 correct walks

Note: ***FT*** their answer to (b)(ii) even if their two vertices are the same.

(iv) D ***A1***

[7 marks]

(c) consider the inequality $e \leq 3v - 6$ for simple planar graphs ***M1***
here, $v = 6$ so $e \leq 12$ ***M1A1***
the graph G has 9 edges so the inequality permits no more than 3 extra edges to be added ***M1AG***

[4 marks]

continued...

Question 4 continued

(d) **METHOD 1**

for G', $v = 6$ and $e = \binom{6}{2} - 9 = 6$ ***A1***

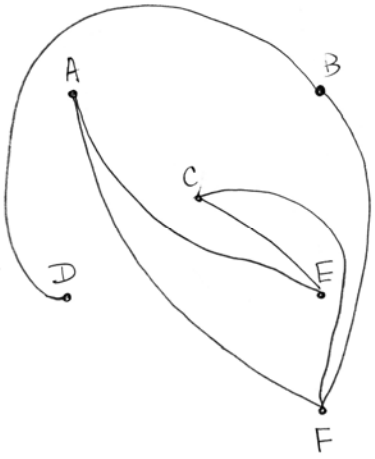
using Euler's relation ***M1***

$f = 2 + e - v$

$= 2$ ***A1***

[3 marks]

METHOD 2



A2

2 faces ***A1***

Note: Only award the final ***A1*** if the previous ***A2*** mark has been awarded.

[3 marks]

Total [18 marks]

5. (a) any clearly indicated method of dividing 1189 by successive numbers ***M1***
find that 1189 has factors 29 and/or 41 ***A2***
it follows that 1189 is not a prime number ***A1***

Note: If no method is indicated, award ***A1*** for the factors and ***A1*** for the conclusion.

[4 marks]

(b) (i) every positive integer, greater than 1, is either prime or can be expressed uniquely as a product of primes ***A1A1***

Note: Award ***A1*** for "product of primes" and ***A1*** for "uniquely".

(ii) **METHOD 1**
let M and N be expressed as a product of primes as follows
$M = AB$ and $N = AC$ ***M1A1***
where A denotes the factors which are common and B, C the disjoint factors which are not common
it follows that $G = A$ ***A1***
and $L = GBC$ ***A1***
from these equations, it follows that
$GL = A \times ABC = MN$ ***AG***

METHOD 2
Let $M = 2^{x_1} \times 3^{x_2} \times ... p_n^{x_n}$ and $N = 2^{y_1} \times 3^{y_2} \times ... p_n^{y_n}$ where p_n denotes the nth prime ***M1***
Then $G = 2^{\min(x_1, y_1)} \times 3^{\min(x_2, y_2)} \times ... p_n^{\min(x_n, y_n)}$ ***A1***
and $L = 2^{\max(x_1, y_1)} \times 3^{\max(x_2, y_2)} \times ... p_n^{\max(x_n, y_n)}$ ***A1***

It follows that $GL = 2^{x_1} \times 2^{y_1} \times 3^{x_2} \times 3^{y_2} \times ... \times p_n^{x_n} \times p_n^{y_n}$ ***A1***
$= MN$ ***AG***

[6 marks]

Total [10 marks]